

$$1. E(r_S) = \sum_{j=1}^3 p(j) r_S(j)$$

$$= .4(.20) + .3(.10) + .3(-.05)$$

$$= .095$$

$$\boxed{E(r_S) = 9.5\%}$$

$$E(r_B) = \sum_{j=1}^3 p(j) r_B(j)$$

$$= .4(.05) + .3(.07) + .3(.04)$$

$$= 0.053$$

$$\boxed{E(r_B) = 5.3\%}$$

$$\sigma_S^2 = \sum_{j=1}^3 p(j) [r_S(j) - E(r_S)]^2$$

$$= .4(.2 - .095)^2 + .3(.1 - .095)^2 + .3(-.05 - .095)^2$$

$$= .100461 + .075(0.01) + .0063075$$

$$= 0.114675$$

$$\rightarrow \sigma_S = 0.107086$$

$$\boxed{\sigma_S = 10.71\%}$$

$$\sigma_B^2 = \sum_{j=1}^3 p(j) [r_B(j) - E(r_B)]^2$$

$$= .4(.05 - .053)^2 + .3(.07 - .053)^2 + .3(.04 - .053)^2$$

$$\sigma_B = .011874$$

$$\boxed{\sigma_B = 1.19\%}$$

2.  $E(r_c)$ ,  $\sigma_c$  ?

$$E(r_c) = r_f + \gamma [E(r_p) - r_f]$$

$$\gamma = \frac{\$3.5 \text{ million}}{\$5.0 \text{ million}}$$

$$\rightarrow \gamma = 0.7$$

$$w_s = \frac{\$2,275,000}{\$3,500,000}$$

$$\rightarrow w_s = 0.65$$

$$w_B = \frac{\$1,225,000}{\$3,500,000}$$

$$\rightarrow w_B = 0.35$$

$$E(r_p) = w_s E(r_s) + w_B E(r_B)$$

$$= 0.65(0.095) + 0.35(0.053)$$

$$= 0.0803$$

$$= E(r_p) = 8.03\%$$

$$\sigma_p^2 = w_s^2 \sigma_s^2 + w_B^2 \sigma_B^2 + 2w_s w_B \text{cov}(r_s, r_B)$$

$$\text{cov}(r_s, r_B) = \sum_{s=1}^3 p(s) [r_s(s) - E(r_s)] [r_B(s) - E(r_B)]$$

$$= 0.4(-.2 - .095)(.05 - .053) + 0.3(-.1 - .095)(.07 - .053) + 0.3(-.05 - .095)(.04 - .053)$$

$$= 4.65e^{-4}$$

$$\sigma_p^2 = (0.65)^2 (0.1071)^2 + (0.35)^2 (0.0119)^2 + 2(0.65)(0.35)(4.65e^{-4})$$

$$= 0.00507517$$

$$\sigma_p = 0.07124$$

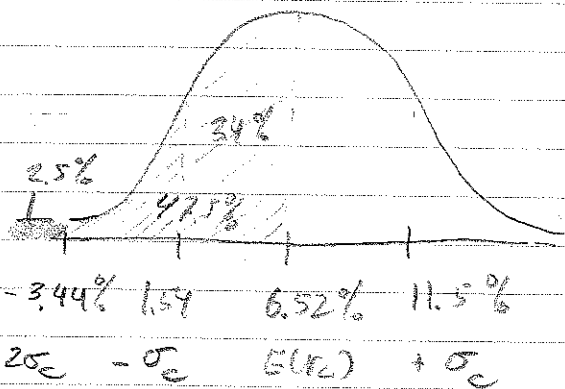
$$\sigma_p = 7.12\%$$

$$\begin{aligned}
 2. \quad E(r_c) &= r_f + y [E(r_p) - r_f] \\
 &= 0.03 + 0.7(0.0803 - 0.03) \\
 &= 0.06521
 \end{aligned}$$

$$\rightarrow \boxed{E(r_c) = 6.52\%} \quad \text{or} \quad \boxed{\$ 326,000}$$

$$\begin{aligned}
 \sigma_c &= y \sigma_p \\
 &= 0.7(0.0712) \\
 &= 0.04984
 \end{aligned}$$

$$\boxed{\sigma_c = 4.98\%}$$



For Client #2, there is a 2.5% chance of losing more than \$ 172,000.

$$3. \quad y = 1.25$$

$$\begin{aligned}
 E(r_c) &= r_{\text{borrow}} + y [E(r_p) - r_{\text{borrow}}] \\
 &= 0.05 + 1.25(0.0802 - 0.05) \\
 &= 0.087875
 \end{aligned}$$

$$\boxed{E(r_c) = 8.79\%}$$

$$\begin{aligned}
 \sigma_c &= 1.25 \sigma_p \\
 &= 0.06225
 \end{aligned}$$

$$\boxed{\sigma_c = 6.23\%} \quad \text{or} \quad 2.5\% \text{ chance of losing more than } \$ 568,000.$$

$$4. \quad W_S = 0.6$$

$$W_B = 0.4$$

$$\sigma_C = 0.05$$

$$\sigma_p^2 = 0.6^2 (0.1071)^2 + (0.4)^2 (0.0119)^2 + 2(0.6)(0.4)(4.65e^{-4})$$

$$\sigma_p^2 = .004375$$

$$\sigma_p = 0.066144$$

$$\sigma_C = y \sigma_p$$

$$y = \frac{5.0000}{6.6144}$$

$$y = 0.755929$$

So, invest 75.59% into the risky portfolio

$$\rightarrow E(r_C) = r_{\text{borrow}} + y[E(r_p) - r_{\text{borrow}}]$$

$$E(r_p) = 0.6(0.095) + 0.4(0.053)$$

$$= 0.0782$$

$$\rightarrow E(r_p) = 7.82\%$$

$$E(r_C) = 0.055 + (1.1416)(0.0782 - 0.05)$$

$$= 0.082193$$

$$\rightarrow E(r_C) = 8.22\%$$

6.  $\beta = 0.65$ ,  $r_m = 0.15$ ,  $r_f = 0.05$

a)  $E(r_i) - r_f = \beta [E(r_m) - r_f]$

$$E(r_i) = 0.05 + 0.65(0.15 - 0.05)$$

$$= 0.115 \text{ or } 11.5\%$$

b)  $P_i^1 = P_i^0 (1 + E(r_i))$

$$= \$100,000 (1.115)$$

$$= \$111,500$$

c)  $\beta < 1$  only reveals that stock is not as volatile as the market. Since it carries less systematic risk, it should not be rewarded with too much return. As long as you hold the adequate portfolio, according to CAPM, BRK-A is already in your portfolio.  $\beta$  is not used to bet on individual stocks.

7.  $E(r_A) = 0.11$      $\beta_A = 0.8$      $r_f = 0.06$      $\sigma_A = 0.10$      $\sigma_M = 0.20$   
 $E(r_B) = 0.14$      $\beta_B = 1.5$      $E(r_m) = 0.12$      $\sigma_B = 0.31$

a) Calculate expected rate of return to each portfolio according to CAPM, then compare to the actual expected rate of return.

$$E(r_A)^{CAPM} = 0.06 + 0.8(0.12 - 0.06)$$

$= 0.108 < 0.11$  ; so actual expected rate of return on A greater than that required by CAPM

$$E(r_B)^{CAPM} = 0.06 + 1.5(0.12 - 0.06)$$

$= 0.15 > 0.14$  ; is less than that required by CAPM on B

In other words, A has positive alpha, so underpriced, add to the portfolio.  $\alpha_A = 0.002$  or 0.2%

b) Invest in the one that gives steeper CAL. The slope of CAL is Sharpe ratio.

$$S_A = \frac{E(r_A) - r_f}{\sigma_A} = 0.5 ; S_B = \frac{E(r_B) - r_f}{\sigma_B} = 0.26 ; \text{ so choose A.}$$